

Thin Domain Walls in Lyra Geometry

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Abstract

This paper studies thin domain walls within the frame work of Lyra Geometry. We have considered two models. First one is the thin domain wall with negligible pressures perpendicular and transverse direction to the wall and secondly, we take a particular type of thin domain wall where the pressure in the perpendicular direction is negligible but transverse pressures are existed. It is shown that the thin domain walls have no particle horizon and the gravitational force due to them is attractive.

1. INTRODUCTION

The topological defects namely domain walls, cosmic strings, monopoles and textures [Kibble(1976); Vilenkin and Shellard(1994)] are formed when the universe underwent a series of phase transitions. In particular, the appearance of domain wall is associated with the breaking of a discrete symmetry i.e. the vacuum manifold M consists of several disconnected components. So, the homotopy group $\pi_0(M)$ is non-trivial [$\pi_0(M) \neq 1$] [Vilenkin and Shellard(1994)]. Hill, Schram and Fry (1989) has suggested that the formation of galaxies are due to domain walls produced during a phase transition after the time of recombination of matter and radiation. Recently the study of the domain walls and space times associated with them has gained renewed cosmological interest due to their application in structure formation in the universe [Pando et al(1998); Vilenkin (1983); Sikivie and Ipser (1984); Schmidt and Wang (1993); Widrow (1989); Chatterji et al (2000); Mukherji (1993)].

Since the discovery of general relativity by Einstein there have been numerous modification of it. Long ago, since 1951, Lyra (1951) proposed an alternating theory of Einstein gravity. He suggested a gauge function into the structureless manifold that bears a close resemblance to Weyl's geometry.

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In the consecutive investigations, Sen (1957) and Sen and Dunn (1971) proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equation based on Lyra geometry, which in normal gauge may be written as

$$R_{ik} - \frac{1}{2}g_{ik}R + \frac{3}{2}\phi_i\phi_k - \frac{3}{4}g_{ik}\phi_m\phi^m = -8\pi GT_{ik} \quad (1)$$

where ϕ_i is the displacement vector and other symbols have their usual meaning as in Riemannian geometry.

According to Halford (1970) the present theory predicts the same effects within observational limits, as far as the classical solar system tests are concerned, as well as tests based on the linearised form of field equations. Soleng (1987) has pointed out that the constant displacement field in Lyra's geometry will either include a creation field and be equal to Hoyle's creation field cosmology or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. Subsequent investigations were done by several authors in scalar tensor theory and cosmology within the framework of Lyra geometry [Bharma (1974); Karadi and Borikar(1978); Beesham (1986); Singh and Singh(1991); Singh and Desikan(1997); Rahaman et al (2001,2002); Casana et al (2005); Rahaman et al (2004)].

In recent, Rahaman (2000,2001,2002,2004) has studied some topological defects within the framework of Lyra geometry.

In this work, we shall deal with thin domain wall, assuming the time like displacement vector

$$\phi_i = [\beta(z, t), 0, 0, 0, 0] \quad (2)$$

And we are looking forward whether the thin domain wall shows any significant properties due the introduction of the gauge field in the Riemannian geometry.

2. The models and the Basic equations

The metric for a plane symmetric space time is taken as

$$ds^2 = e^A(dt^2 - dz^2) - e^C(dx^2 + dy^2) \quad (3)$$

where $A = A(z, t); C = C(z, t)$.

The energy stress components in co-moving coordinates for the thin domain wall under consideration here are given by

$$T_t^t = \rho, T_x^x = T_y^y = p_1, T_z^z = 0, T_t^z = 0. \quad (4)$$

where ρ is the energy density of the wall, p_1 is the tension along X and Y directions in the plane of the wall and pressure in the perpendicular direction to the wall is negligible. The stress tensors $T_x^x = T_y^y = p_1$ corresponding to the tension of the wall along X and Y directions are assumed to be zero in the **First case** and $T_x^x = T_y^y = T_t^t = p_1 = \rho$ in the **Second case**.

The field equations for the metric (2) are

$$\frac{e^{-A}}{4}[2A'C' - 4C'' - 3(C')^2] + \frac{e^{-A}}{4}[(\dot{C})^2 + 2\dot{A}\dot{C}] - \frac{3}{4}\beta^2 e^{-A} = 8\pi\rho \quad (5)$$

$$\frac{e^{-A}}{4}[4\ddot{C} + 3\dot{C}^2 - 2\dot{A}\dot{C}] + \frac{e^{-A}}{4}[-(C')^2 - 2A'C'] + \frac{3}{4}\beta^2 e^{-A} = 0 \quad (6)$$

$$\frac{e^{-A}}{4}[-2A'' - 2C'' - (C')^2] + \frac{e^{-A}}{4}[2\ddot{C} + 2\ddot{A} + (\dot{C})^2] + \frac{3}{4}\beta^2 e^{-A} = 8\pi p_1 \quad (7)$$

$$\frac{1}{2}[\dot{C}' + \dot{C}(A' - C') + \dot{A}C'] = 0 \quad (8)$$

[" ." and " ' " denotes the differentiation w.r.t. t and z respectively .]

3. Solutions

Case - I : $p_1 = 0$

To solve the field equations, we shall assume the separable form of the metric coefficients as follows :

$$A = A_1(z) + A_2(t); C = C_1(z) + C_2(t); \quad (9)$$

From equation (8), by using separable form, we get,

$$\frac{C_1' - A_1'}{C_1'} = \frac{\dot{A}_2}{\dot{C}_2} = 1 - m \quad (10)$$

where $(1 - m)$ is the separable constant.

This implies,

$$A_1 = mC_1 \quad (11)$$

$$A_2 = (1 - m)C_2 \quad (12)$$

From eqn. (6) and eqn. (7) and by using eqn. (9) and (11), we get

$$2mC_1'^2 - (2m+2)C_1'' = 2m\ddot{C}_2 + 2m\dot{C}_2^2 = n \quad (13)$$

(n being the separation constant) .

Solving eqn.(13), we get

$$C_1 = \frac{1}{b} \ln \sinh(abz) \quad (14)$$

where $a^2 = \frac{n}{2m}$ and $b = \frac{m}{m+1}$.

For time part , we get

$$C_2 = \ln \cosh(at) \quad (15)$$

So, finally the complete solutions for the metric coefficients may be expressed in the form

$$e^A = [\sinh(abz)]^{\frac{m}{b}} [\cosh(at)]^{1-m} \quad (16)$$

$$e^C = [\sinh(abz)]^{\frac{1}{b}} [\cosh(at)] \quad (17)$$

The energy of the wall is

$$8\pi\rho = \frac{a^2}{[\sinh(abz)]^{\frac{m}{b}} [\cosh(at)]^{1-m}} \left[1 + \frac{b}{\sinh(abz)^2} - \coth(abz)^2 \right] \quad (18)$$

Here $\beta^2(z, t)$ takes the following form,

$$\beta^2(z, t) = \frac{1}{3} \left[a^2 [\coth(abz)]^2 - \frac{(2m+2)a^2b}{\sinh(abz)^2} - a^2 \tanh(at)^2 - \frac{2(2-m)a^2}{\cosh(at)^2} \right] \quad (19)$$

Case - II :

Here we construct another model of a thin domain wall.

We assume that $T_x^x = T_y^y = T_t^t = \rho$.

In view of the above forms of energy stress tensors and using field equations, we find the following solutions

$$e^A = [\sinh(DBz)]^{\frac{m}{B}} [\cosh(Dt)]^{1-m} \quad (20)$$

$$e^C = [\sinh(DBz)]^{\frac{1}{B}} [\cosh(Dt)] \quad (21)$$

where $D^2 = \frac{N}{2(m+2)}$; $B = \frac{m+2}{m-1}$ and N = separation constant .

The energy of the wall is

$$8\pi\rho = \frac{D^2}{[\sinh(DBz)]^{\frac{m}{B}} [\cosh(Dt)]^{1-m}} \left[1 + \frac{B}{\sinh(DBz)^2} - \coth(DBz)^2 \right] \quad (22)$$

Here $\beta^2(z, t)$ takes the following form,

$$\beta^2(z, t) = \frac{1}{3} [D^2(2m+1)[\coth(DBz)]^2 - D^2(2m+1)\tanh(Dt)^2 - \frac{4D^2}{\cosh(Dt)^2}] \quad (23)$$

We see that the above solution take the same form as in **Case-I** except for $m = 1$. The model in **Case- I** exists for $m = 1$ where as the model in **Case-II** does exist for $m \neq 1$. Therefore the nature of the solutions of the above two models should be the same except for $m = 1$.

4. Discussions :

From the results given above, it is evident that at any instant the domain wall density, ρ decreases with the increase of the distance from the symmetry plane (both sides of the symmetry plane) and ρ vanishes as $z \rightarrow \pm\infty$.

The general expression for the three space volume is given by

$$\sqrt{|g_3|} = [\sinh(abz)]^{\frac{m+2}{2b}} [\cosh(at)]^{\frac{3-m}{2}} \quad (24)$$

Thus the temporal behaviour would be

$$\sqrt{|g_3|} \sim [\cosh(at)]^{\frac{3-m}{2}} \quad (25)$$

If $m > 3$, then the three space collapses . On the other hand when $m < 3$, there are expansion along Z-direction .

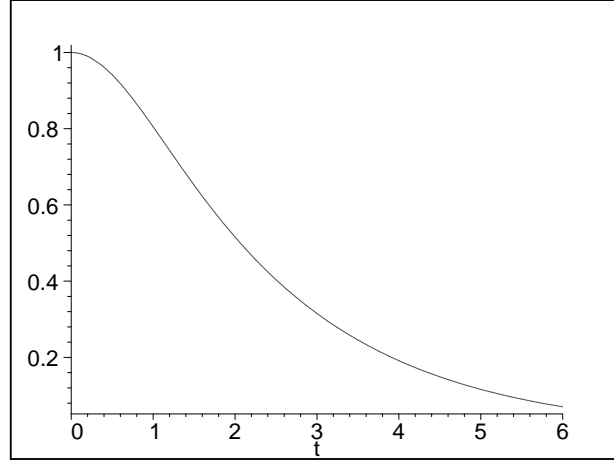


Figure 1: The temporal behaviour of three space volume for $m > 3$

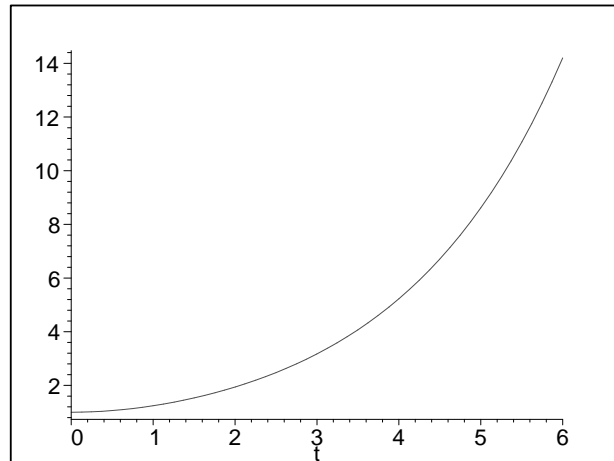


Figure 2: The temporal behaviour of three space volume for $m < 3$

Similar results exist for **Case-II**.

For $m = 1$, the energy stress components are time independent whereas metric itself depends on time. This is similar to Goetz's domain wall [Goetz(1990)].

We now calculate the proper distance, S_H (between $z = 0$, the centre of the wall and $z \rightarrow \infty$) measured along a space like curve running perpendicular to the wall($t, x, y = \text{Const.}$) as

$$S_H = \int \exp\left(\frac{A}{2}\right) dz = [\cosh(at)]^{\frac{1-m}{2}} \int [\sinh(abz)]^{\frac{m}{2b}} dz \quad (26)$$

This distance diverges.

Hence there is no horizon in the Z-direction i.e. perpendicular to the wall. This is very similar to the result obtained by Wang (Wang, 1994) for a Thick Wall in Einstein's theory but at variance with Goetz's result.

The repulsive and attractive character of the wall can be discussed by either studying the time like geodesic in the space time or analyzing the acceleration of an observer who at rest relative to the wall.

Let us consider an observer with four velocity given by

$$V_i = [\sinh(abz)]^{\frac{m}{2b}} [\cosh(at)]^{\frac{1-m}{2}} \delta_i^t \quad (27)$$

Then we obtain the Acceleration Vector

$$A^i = V_{;k}^i V^k = \frac{am}{2} \frac{\coth(abz)}{[\sinh(abz)]^{\frac{m}{b}} [\cosh(at)]^{1-m}} \quad (28)$$

It is evident that A^i is positive and it follows that an observer who wishes to remain stationary with respect to the wall must accelerate away from the wall. In other words, the wall exhibits an attractive nature to the observer. Similar conclusion can be drawn for the domain wall solution in **Case - II**. This result is also conformity with Wang [Wang, (1994)] but differs from Goetz [Goetz(1990)]. We are surprising to note that the displacement vector still exist after infinite time. For a future exercise, it will be interesting to study different properties of different Topological Defects within the frame work of Lyra Geometry.

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